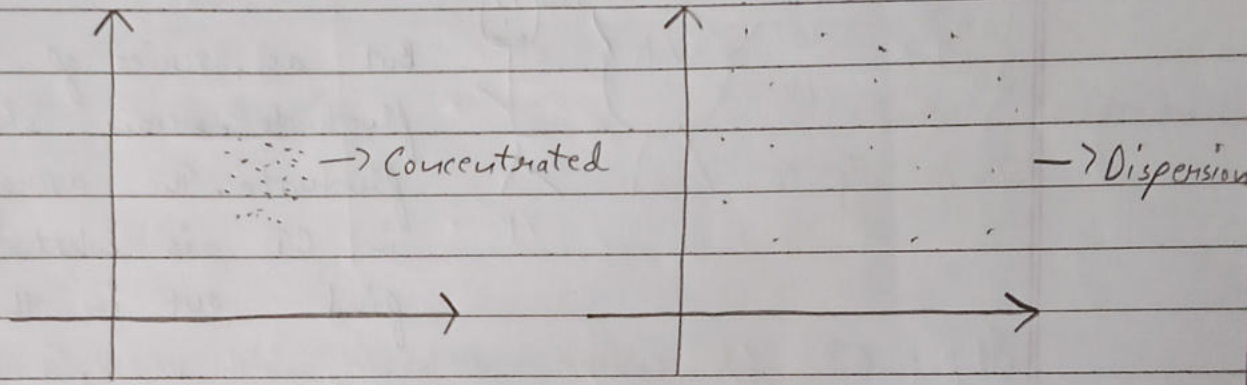
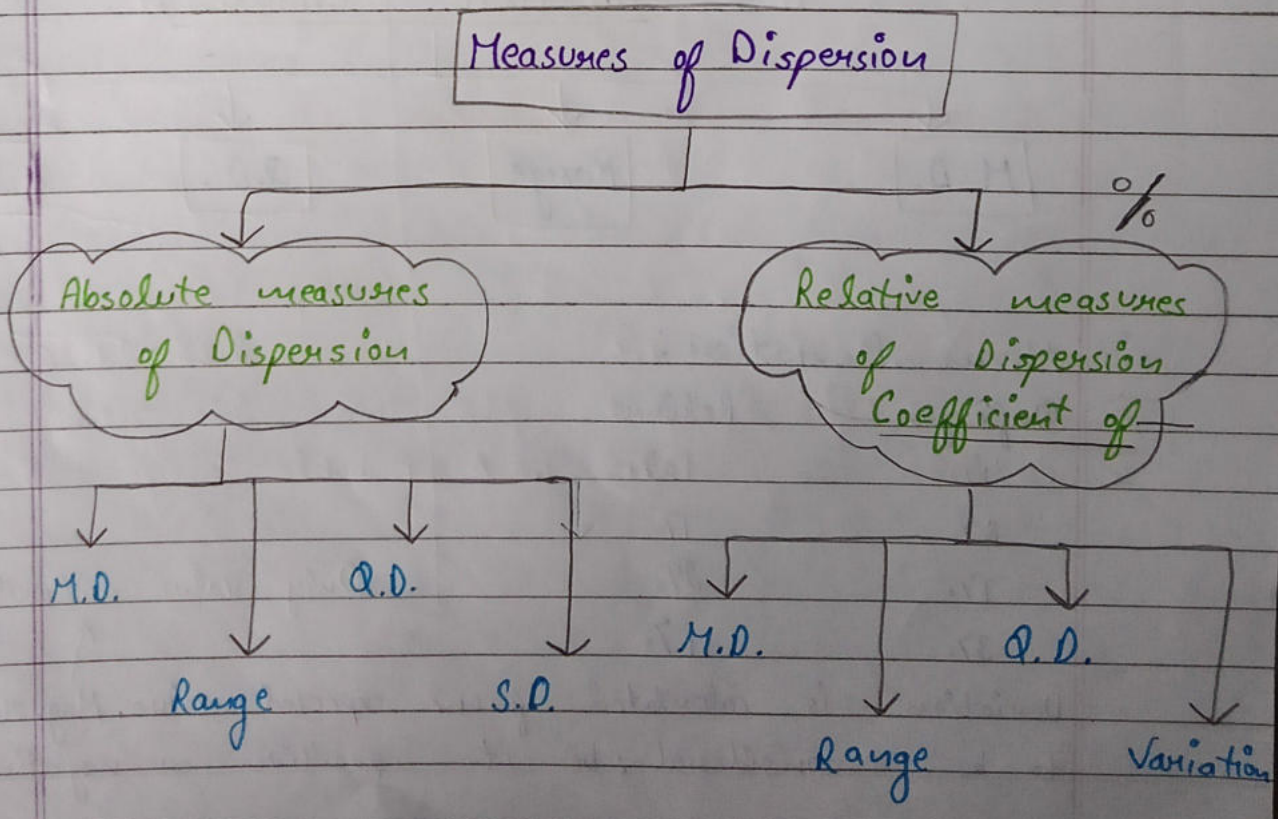
Unit -2

Dispersion

What is meant by dispersion?

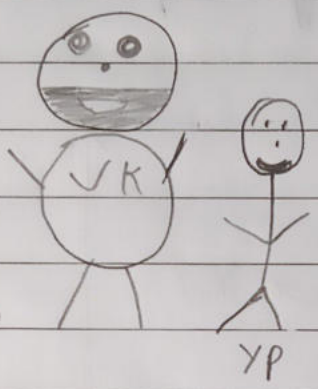


Scatteredness of data is called dispersion.



Why MOD is referred as second order of averages?

Less Consistent (x) Yusuf Pathan	(y) Virat Kohli (More consistent)
150	70
0	70
100	70
30	70
$\bar{x} = 70$	$\bar{y} = 70$



Mean → Expected value → We believe ki agla observation CT ke aas pas hoga.

But as, series of observations fluctuate, mean should also fluctuate. The average fluctuation in CT is what we should find out in the chapter.

CT ± MOD

→ avg. fluctuation in CT

ABSOLUTE MEASURES

- M.D.
- Range
- Q.D.
- S.D.

(i) Mean Deviation:

Infosys	Reliance
4%	10%
8%	7%
7%	9%
3%	11%

Only value without consider sign

Deviation is calculated from expected value. Magnitude has to be considered; be it positive or negative deviation.

F.D. for 1 year

Bank 1,00,000 → 1,08,000

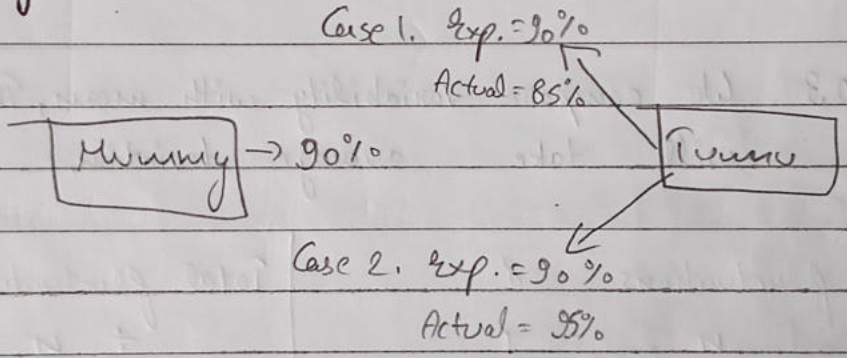
RIL 1,00,000 → 1,50,000

→ 40,000

Fluctuation ↑ MOD ↑ RISK ↑

Deviation = |Actual − Expected| ..→ Direct Relation

Story time



★ 3 Steps to calculate mean deviation :

Step-01 Calculate deviation from $\bar{x}$ / x_{mo} / x_{md}

Infosys (x)	Actual − Expected $(x - \bar{x})$	Reliance (y)	Actual − Expected $(y - \bar{y})$
4	−1.5	10	0.75
8	2.5	7	−2.25
7	1.5	9	−0.25
3	−2.5	11	1.75
	0		0

$\bar{x} = 5.5\%$

$\bar{y} = 9.25$

Step -02 We need absolute deviation. Therefore, use modulus.

Actual - Expected $(x - \bar{x})$	$\lvert x - \bar{x} \rvert$
-1.5	1.5
2.5	2.5
1.5	1.5
-2.5	2.5
0	8

Actual - Expected $(y - \bar{y})$	$\lvert y - \bar{y} \rvert$
0.75	0.75
-2.25	2.25
-0.25	0.25
1.75	1.75
0	5

Step - 03 We compare variability with mean, Therefore, we will take average deviation.

Total fluctuations = 8
÷ n = 4
= 2
↓ Mean Deviation

Interpretation →
Max. 5.5 + 2 = 7.5
Min. 5.5 - 2 = 3.5
Probable range = 3.5 - 7.5 %

Total fluctuations = 5
÷ n = 4
= 1.25
↓ Mean Deviation

Interpretation →
Max. 9.25 + 1.25 = 10.5
Min. 9.25 - 1.25 = 8
Probable range = 8 - 10.5 %

Final Formula of mean deviation :

$$M.D. = \frac{\Sigma |x - \bar{x}| / x_{mo} / md|}{n}$$

* Unless otherwise stated M.D. is calculated from mean.

For eg. Calculate M.D.

X	$(x - \bar{x})$	$\lvert x - \bar{x} \rvert$
2	-3	3
4	-1	1
6	1	1
8	3	3
	0	8

$\bar{x} = 5$

$MD = \frac{8}{4} = 2$

Max. $5 + 2 = 7$

Min. $5 - 2 = 3$ 7-3 %

D. O. C. →

Type $2+4+6+8 = \div 4 = 5$

Type 2-5 M-
4-5 M-
6-5 M+
8-5 M+
} MRC ÷ 4

Eg. Calculate mean deviation from median.

x: ~~7, 8, 6, 7~~, 12, 14, ~~17, 18, 15, 26~~

13

$x_{md} = 13$

$MD = \frac{\Sigma |x - x_{md}|}{n}$

$= \frac{62}{10} = 6.2$

Eg. Calculate M.D. of,

x: 4, 9, 7, 6, 10, 12, 8

$MD = \frac{\Sigma |x - \bar{x}|}{n}$

$= \frac{14}{7} = 2$

(ii) Range: "It is the maximum area within which a variable can dance."

It is the quickest measures of dispersion.

Range = Maximum - Minimum

Infosys

Min. 2%

Max. 15%

Range = 15 - 2

= 13%

Reliance

Min. 5%

Max. 40%

Range = 40 - 5

= 35%

Eg. 2, 8, 10, 7, 6, 90, 94, 80, 16, 2, 8
Calculate Range

Max. Value = 94
Min. Value = 2

Range = 94 - 2
= 92 Ans

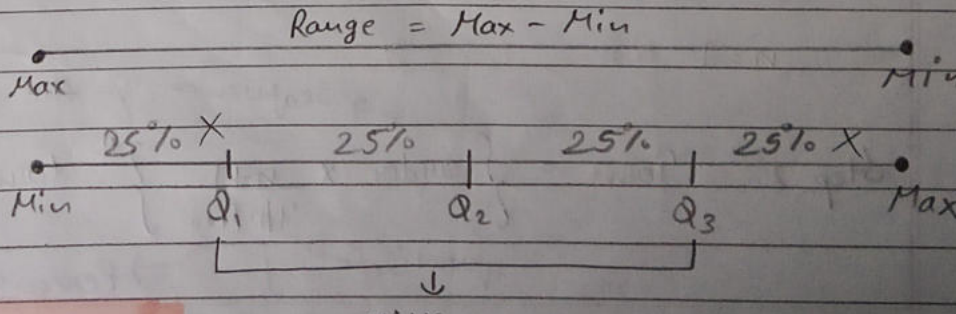
★ Limitation:

1. It is highly inaccurate & may lead to misleading conclusions.

V.K. → 80, 85, 92, 83, 0, 150, 76, 84

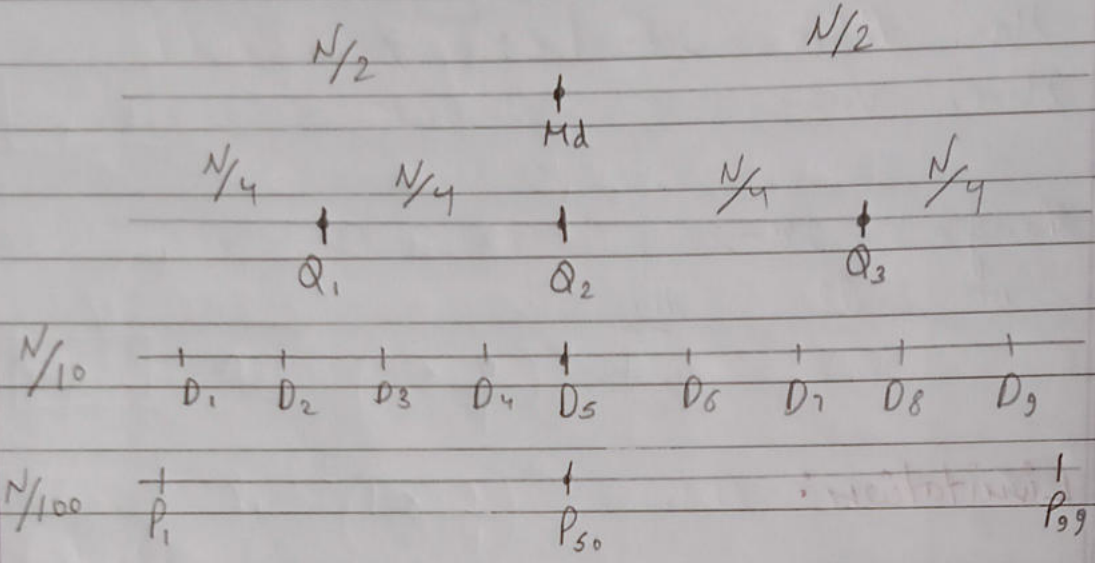
Range = 150 - 0 = 150
The problem with range is that it concludes V.K. is highly inconsistent but the data shows otherwise.

(iii) Quartile Deviation : Unlike range, it is not affected by extremities of the observation.



Q.D. = $\frac{Q_3 - Q_1}{2}$

→ Concept of Partition Values



$$\boxed{Md = Q_2 = D_5 = P_{50}}$$

x: 2, 7, 18, 9, 6, 15, 4, 3, 21

Calculate median, D_5, Q_3, P_{77}

Step 1 Arrange in increasing order.

2, 3, 4, 6, 7, 9, 15, 18, 21

$n = 9$

Step 2 Term = { order × $\frac{n+1}{4/10/100}$ }th term

order → sequence of the partition value

4 → quartiles

10 → Deciles

100 → Percentiles

$$D_5 = \left(5 \times \frac{9+1}{10}\right)^{th} \text{ term} = 5^{th} \text{ term}$$

$$Q_3 = \left(3 \times \frac{9+1}{4}\right)^{th} \text{ term} = 7.5^{th} \text{ term}$$

$$P_{77} = \left(77 \times \frac{9+1}{100}\right)^{th} \text{ term} = 7.7^{th} \text{ term}$$

Step 3 Value of the term = Base term + (aage wali term − Base term) × decimal Part

$D_5 = 5^{th}$ term = 7 Ans

$Q_3 = 7.5^{th}$ term = VII + (VIII − VII) × 0.5

$Q_3 = 15 + (18-15) \times 0.5 = 16.5$ Ans

$P_{77} = 7.7^{th}$ term = VII + (VIII − VII) × 0.7

$P_{77} = 15 + (18-15) \times 0.7 = 17.1$ Ans

For Eg. Calculate Q.D.

x : 2, 4, 8, 10, 12, 16, 4, 9

Step 1 : Increasing Order

2, 4, 4, 8, 9, 10, 12, 16

$n = 8$

Step 2 : $Q_3 = \left(3 \times \frac{9}{4}\right)^{th}$ term $= 6.75$

$Q_1 = \left(1 \times \frac{9}{4}\right)^{th}$ term $= 2.25$

Step 3 : $Q_3 = 6.75^{th}$ term

$= \overline{VI} + (\overline{VII} - \overline{VI}) \times 0.75$

$= 10 + (12 - 10) \times 0.75$

$= 11.5$

$Q_1 = 2.25^{th}$ term

$= \overline{II} + (\overline{II} - \overline{III}) \times 0.25$

$= 4 + (4 - 4) \times 0.25$

$= 4$

Step 4 $Q.D. = \frac{Q_3 - Q_1}{2}$

$Q.D. = \frac{11.5 - 4}{2}$

$\boxed{Q.D. = 3.75}$ Ans

★ $Q_3 - Q_1$ = Inter quartile Range

★ $\frac{Q_3 - Q_1}{2}$ = Semi - inter quartile range

(iv) Standard Deviation

Britania - Company

Year	Returns
2001	10%
2002	12%
2003	18%
2004	16%
2005	14%

$\bar{x} = \frac{\Sigma x}{n}$

$\bar{x}$ = Average Return = $\frac{10 + 12 + 18 + 16 + 14}{5}$

$\bar{x} = 14\%$

Step. 01 Calculate deviation from A.M.

x:	10%	12%	18%	16%	14%
$x - \bar{x}$:	-4%	-2%	+4%	+2%	0%

Step. 02 Calculate sum of squares of deviation:

$x - \bar{x}$:	-4%	-2%	+4%	+2%	0%
$(x - \bar{x})^2$:	$16\%^2$	$4\%^2$	$16\%^2$	4%	$0\%^2$

$\Sigma (x - \bar{x})^2 = 40\%^2$

Step. 03 Calculate average deviation in right unit.

Total deviation = $\Sigma (x - \bar{x})^2 = 40\%^2$

$\div n$ $\div 5$

$8\%^2$ → Variance

$\sqrt{\frac{\Sigma (x - \bar{x})^2}{n}} = \sqrt{8\%^2} = 2.82\%$

↳ Standard deviation

Final Formula

$$\text{Standard deviation} = \sqrt{\frac{\Sigma (x - \bar{x})^2}{n}}$$

$$\text{Variance} = \frac{\Sigma (x - \bar{x})^2}{n}$$

Eg. Calculate S.D. of following stocks :

Infosys	Reliance
80%	10%
85%	12%
70%	08%
75%	04%

→ Infosys

(x)	$(x-\bar{x})$	$(x-\bar{x})^2$
80	2.50	6.25
85	7.50	56.25
70	-7.50	56.25
75	-2.50	6.25
		125

$\bar{x} = 77.50$

$\text{Variance} = \frac{\Sigma (x-\bar{x})^2}{n} = \frac{125}{4} = 31.25$

$S.D. = \sqrt{\frac{\Sigma (x-\bar{x})^2}{n}} = \sqrt{31.25\%^2}$

$= 5.59\%$

D.O.C. →

Type 80 + 85 + 70 + 75 ÷ 4

Type 80 - 77.50 X = M+

85 - 77.50 X = M+

70 - 77.50 X = M+

75 - 77.50 X = M+

MRC ÷ 4 = √

Reliance

(y)	$(y-\bar{y})$	$(y-\bar{y})^2$
10	1.50	2.25
12	3.50	12.25
08	-0.50	0.25
04	-4.50	20.25
		35

$\bar{y} = 8.50$

$\text{Varian} = \frac{\Sigma (y-\bar{y})^2}{n} = \frac{35}{4} = 8.75$

$S.D. = \sqrt{\frac{\Sigma (y-\bar{y})^2}{n}} = \sqrt{8.75\%^2}$

$= 2.95\%$

D.O.C. →

Type 10 + 12 + 08 + 04 ÷ 4

Type 10 - 8.50 X = M+

12 - 8.50 X = M+

08 - 8.50 X = M+

04 - 8.50 X = M+

MRC ÷ 4 = √

Shortcut Formula

$$S.D. = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2}, \quad Var = \frac{\Sigma x^2}{n} - (\bar{x})^2$$

Infosys

x	x^2
80	6400
85	7225
70	4900
75	5625
	24150

$\bar{x} = \frac{80+85+70+75}{4} = 77.50$

$S.D. = \sqrt{\frac{24150}{4} - (77.5)^2}$

$S.D. = 5.59$

D.O.C. → $\frac{24150}{4}$ M+

77.5^2 M−

MRC √

Reliance

y	y^2
10	100
12	144
08	64
04	16
	324

$\bar{y} = \frac{10+12+8+4}{4} = 8.5$

$S.D. = \sqrt{\frac{324}{4} - (8.5)^2}$

$S.D. = 2.95$

D.O.C. → $\frac{324}{4}$ M+

8.5^2 M−

MRC √

□ Properties of S.D.

(1) It is the most popular measure of dispersion.

(2) There is a shortcut formula;

Applicability → Jab bhi koi balancing figure nikalna ho, toh always use shortcut formula

$$S.D. = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2}$$

(3) SD^2 = Variance

σ^2 = Var (x) OR σ = $\sqrt{\text{Variance}}$

Q. Calculate variance if S.D. is 8.

S.D. = 8

Variance = $SD^2 = 8^2$

= 64

Q. If variance is 81, how much is S.D.?

$\sqrt{\text{Var}}$ = S.D.

$\sqrt{81}$ = 9 Ans.

(4) S.D. is represented by आधा σ।

σ → Sigma

(5) S.D. of first n natural no's:

$$S.D. = \sqrt{\frac{n^2-1}{12}}$$

0, 1, 2, 3, 4, 5 → No

1, 2, 4, 5, 6 → No

2, 3, 4, 5, 6 → No

1, 2, 3, 4, 5, 6 → Yes

E.g. Calculate S.D. of 1, 2, 3, 4, 5, 6

$$S.D. = \sqrt{\frac{n^2-1}{12}} = \sqrt{\frac{35}{12}}$$

S.D. = 1.7078

Eg. Calculate S.D. of 1, 2, 3, 4.

$$S.D. = \sqrt{\frac{n^2-1}{12}} = \sqrt{\frac{15}{12}}$$

S.D. = 1.1180

(6) S.D. between 2 numbers is $\frac{|a-b|}{2}$ or $\frac{|\text{difference of 2 no.'s}|}{2}$ or half of range. (Also applicable on M.D.)

No M.D. is negative
S.D., M.D., Q.D., Range are always +ve.

Eg. Calculate S.D. of 8 & 10

$$S.D. = \frac{|10-8|}{2} = \frac{2}{2} = 1$$

(7) If all the observations are repeated for the same no. of times then count it only once.

Eg. 2, 5, 7, 5, 7, 2, 5
No

(It is also applicable on M.D.)

Eg. Calculate S.D. of 2, 3, 4, 4, 3, 2 Yes

S.D. of 2, 2, 3, 3, 4, 4 = S.D. of 2, 3 & 4

S.D. (σ) → Type 2 + 3 + 4 ÷ 3 = 3
Type 2 − 3 × = M+
Type 3 − 3 × = M+
Type 4 − 3 × = M+
Type MRC ÷ 3
Type = √

S.D. = 0.8164

Ans

(8) S.D. has desirable mathematical properties. It means combines S.D. can be calculated.

$$\sigma_{12} = \sqrt{\frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}}$$

n_1 = no. of observations of group 1
n_2 = no. of observations of group 2
σ_1 = S.D. of group 1
σ_2 = S.D. of group 2
$d_1 = \bar{x}_{12} - \bar{x}_1$
$d_2 = \bar{x}_{12} - \bar{x}_2$

e.g. Calculate Combined S.D.

	Infosys (x_1)		Reliance (x_2)
2001	10%	2001	10%
2002	12%	2002	12%
2003	18%	2003	8%
2004	12%		
n_1	= 4	n_2	= 3
σ_1	= 3	σ_2	= 1.6329
$\bar{x}_1$	= 13	$\bar{x}_2$	= 10

$$\bar{x}_{12} = \frac{n_1\bar{x} + n_2\bar{x}_2}{n_1 + n_2} = \frac{4 \times 13 + 3 \times 10}{7}$$

$$\bar{x}_{12} = 11.71$$

$d_1 = \bar{x}_{12} - \bar{x}_1$

$d_1 = 11.71 - 13$

$d_1 = -1.29$

$d_2 = \bar{x}_{12} - \bar{x}_2$

$d_2 = 11.71 - 10$

$d_2 = 1.71$

$$\sigma_{12} = \sqrt{\frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}}$$

$$\sigma_{12} = \sqrt{\frac{4(3^2 + (1.29)^2) + 3(1.6329^2 + 1.71^2)}{4 + 3}}$$

$$\sigma_{12} = \sqrt{\frac{4(9 + 1.66) + 3(2.66 + 2.92)}{7}}$$

$$\sigma_{12} = \sqrt{\frac{42.64 + 16.74}{7}}$$

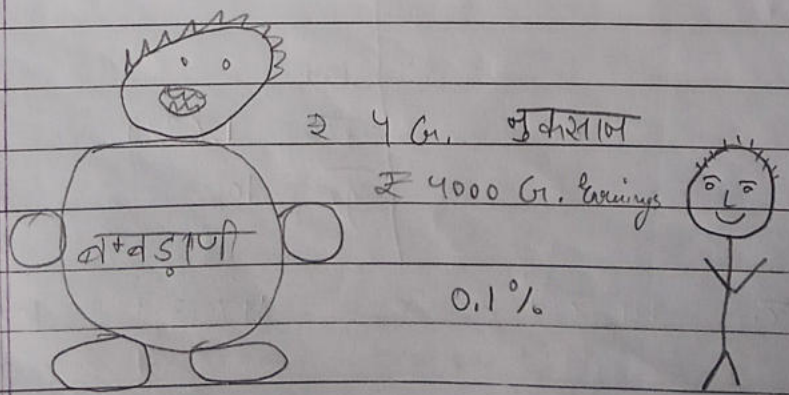
$$\sigma_{12} = \sqrt{\frac{59.38}{7}}$$

$$\sigma_{12} = \sqrt{8.48}$$

$$\boxed{\sigma_{12} = 2.91}$$ Ans

Relative Measures of Dispersion – Wherever comparison is involved, always use relative M.O.D. (Bhale hi Que. Chupki ho)

Concept:



Relative Measures

Coefficients

(i) Coefficient of M.D.

$$M.D. = \frac{\Sigma |x - \bar{x}/x_{mo}|x_{md}|}{n} \rightarrow \text{Risk}$$

Mean / Mode / Median = Return

$$\text{Coefficient of M.D.} = \frac{\text{Risk}}{\text{Return}} \times 100 \;,\; \frac{M.D.}{\bar{x}|x_{mo}|x_{md}} \times 100$$

E.g. Decide, which is a more stable stock?

Years	Infosys		Years	Reliance	
2001	8%	$\bar{x} = 10.5$	2001	18%	$\bar{y} = 11.5$
2002	12%	M.D. = 3.5	2002	12%	M.D. = 3.5
2003	16%		2003	9%	
2004	6%		2004	7%	

Coefficient of M.D. $= \frac{3.5}{10.5} \times 100$

$= 33.33\%$

Coefficient of M.D. $= \frac{3.5}{11.5} \times 100$

$= 30.43\%$

Lower the Better !

We will select RIL as it has lower Coff. of M.D

(ii) Coefficient of range

It explains risk as a % of returns.

$$\text{Coefficient of risk} = \frac{\text{Avg. risk}}{\text{Avg. return}} \times 100$$

$$\text{Coefficient of Range} = \frac{\text{Max} - \text{Min}}{\text{Max} + \text{Min}} \times 100$$

Eg. Calculate coefficient of range.

x: 2, 4, 6, 10, 18, 7, 9, 26

Max. = 26

Min. = 2

$$\text{Coefficient of range} = \frac{26-2}{26+2} \times 100$$

$$= \frac{24}{28} \times 100$$

$$= 85.71\ \%$$

(iii) Coefficient of Quartile Deviation

$$\text{Coefficient of Q.D.} = \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$$

Eg. Calculate coefficient of Q.D.:

$Q_1 = 80 \qquad Q_3 = 120$

$$\text{Coefficient of Q.D.} = \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$$

$$= \frac{120 - 80}{120 + 80} \times 100$$

$$= 20\%$$

Other way

$$\text{Coefficient of Q.D.} = \frac{\frac{Q_3 - Q_1}{2}}{\frac{Q_3 + Q_1}{2}} \times 100$$

$$= \frac{Q.D}{Q_2 = Md} \times 100$$

$$\text{Coefficient of Q.D.} = \frac{Q.D.}{Md} \times 100$$

(iv) Ø Coffiicient of variation

It is risk as a % of return

Risk = S.D.
Return = $\bar{x}$

Coefficient of variation = $\frac{\sigma}{\bar{x}} \times 100$

Decision Rule: "Lower it is, the better it is"

It is a measure of risk showing it as a % of return.

☞ Whenever, consistency is asked always calculate S.D. If consistency needs to be compared; always calculate coefficient of variation (chupki)

Eg. Find out that which is the more consistent stock?

Infosys		Reliance	
80%	$\bar{x} = 77.5$	10%	$\bar{y} = 8.5$
85%	S.D. = 5.590	12%	S.D. = 2.958
70%		08%	
75%		04%	

Coff. of variation = $\frac{\sigma}{\bar{x}} \times 100$

$= \frac{5.590}{77.5} \times 100$

$= 7.21\%$

Coff. of variation = $\frac{\sigma}{\bar{y}} \times 100$

$= \frac{2.958}{8.5} \times 100$

$= 34.8\%$

Infosys is the more consistent stock

Eg. Who is the more consistent batter?

Virat Kohli		Yusuf Pathan	
80	$\bar{x} = 65$	10	$\bar{y} = 40$
75	S.D. = 39.49	12	S.D. = 55.07
120		18	
0		150	
50		10	
$C.V. = \frac{\sigma}{\bar{x}} \times 100$		$C.V. = \frac{\sigma}{\bar{y}} \times 100$	
$= \frac{39.49}{65} \times 100 = 60.75\%$		$= \frac{55.07}{40} \times 100 = 137.67\%$	
Decision: More Consistent		Less Consistent	

लावारिस Property 2.0 :→ It is applicable on all the MOD (SD, Range, QD, MD)

	C ↑	MOD
△ of origin	✓	X
△ of scale	✓	✓
△ of sign	✓	X

x	2x	x/2	x+2	x−2	−x
		△ of scale	△ of origin		△ of sign
2	4	1	4	0	−2
4	8	2	6	2	−4
6	12	3	8	4	−6
8	16	4	10	6	−8
$\sigma = 2.23$	$\sigma = 4.46$	$\sigma = 1.11$	$\sigma = 2.23$	$\sigma = 2.23$	$\sigma = 2.23$

If all the observations are affected by scale, MOD's are similarly affected.

For Eg. S.D. of x is 4. The relationship b/w x & y is given as $2x + 4y = 8$. Calculate S.D. of y.

Step 1. Puri dhara ek taraf, jisko dundh rahe hai vo ek taraf.

Step 2. Ignore origin & sign & only consider scale & put the value.

$$2x + 4y = 8 \Rightarrow 4y = -2x + 8$$

$$y = \frac{-2x}{4} + \frac{8}{4}$$

~~$y = \frac{-x + 2}{2}$~~

$$\sigma_y = \frac{2}{4} \times 4$$

$$\sigma_y = 2 \quad \text{Ans}$$

Eg. $2x + 9y = 7$

Calculate MD_y if M.D. of x is 8

$$2x + 9y = 7 \Rightarrow y = \frac{-2x}{9} + \frac{7}{9}$$

$$y = \frac{2}{9} \times 8$$

$$\boxed{y = 1.77}$$

E.g. x : 2 4 6 8 S.D. of x is 4. Find σ_y

y : -3 -1 1 3

~~$2x - 3y$~~ $y = x - 5$

$\sigma_y = \sigma_x$

$\boxed{\sigma_y = 4}$

✓ Relationship b/w S.D., Q.D. & M.D.

"4, 5, 6 karke samundar me kudo"

4 S.D. = 5 M.D. = 6 Q.D.

↓ ↓ ↓

Samundar me kudo

Q. S.D. = 8 Find Q.D.

4 S.D. = 6 Q.D.

$\frac{4 \times 8}{6}$ = Q.D. $\Rightarrow$ Q.D. = 5.333 Ans

सावधान Property is N.A. on:

1) Variance
2) Relative MOD